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Extraction of Low Mobility, Low Conductivity Carriers from Field Dependent Hall Data

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Hall measurements are used to measure the mobility and density of carriers in semi-conductors. Field dependent Hall measurements coupled with Quantitative Mobility Spectrum Analysis (QMSA) can be used to determine the number of carriers and the mobility and density of each carrier. The utility of the QMSA technique is lacking however if very low mobility carriers are present. Various least squares fitting methods have been used over the years for analyzing variable field Hall data. In this paper two different methods for least squares fitting are compared to determine which methods work best for low mobility, low conductivity carriers.

1. Introduction

A method, based on multi-carrier methods, of analyzing field dependant Hall effect data has been developed. This procedure allows the determination of low mobility, low conductivity carriers in materials with high conductivity carriers as well as low conductivity carriers. Methods like QMSA[1] can require unrealistic high fields to resolve low mobility carriers, for instance for a carrier mobility of 1 cm^2/V s, ideally the magnetic field should be 10^4 T. Typical methods of least squares fitting of Hall data used simplex methods or trust region methods[2]. The method described here uses constrained linear fits to the conductivity[3]. The constraints are such that the conductivity is required to be positive. It is found that non-negative least squares work much better then methods using a “brick wall” in the mobility fit if the conductivity is negative. This linear fit was used with a systematic search of the mobility space for local minimum of the chi squared. This local minimum was then used to start a non-linear constrained least squares solution. The constraints in this case were to keep the signs of the mobility fixed.

2. Theoretical background

For a single carrier material, the measured Hall coefficient and resistivity are given by,[4]

$$R_H(B) = \frac{1}{nq} \qquad (1)$$

$$\rho(B) = \frac{1}{nq\mu}, \qquad (2)$$

and the conductivity tensor is given by,

$$\sigma_{xx} = \frac{nq\mu}{1+\mu^2 B^2} \qquad (3)$$

$$\sigma_{xy} = \frac{nq\mu^2 B}{1+\mu^2 B^2}. \qquad (4)$$

Here n is the carrier concentration, μ is the mobility, q is the charge of the carrier, and B is the magnetic field. It is apparent from Eqs. (1) and (2) that for single carrier materials the Hall coefficient and resistivity are field independent. The single field Hall characterization is therefore sufficient for such materials.

However, for multi-carrier systems the mobility and density calculated from Eqs. (1) and (2) will be averaged over all carriers. For such materials, conductivities of the individual carriers are additive, and the total conductivity tensor (of a N-carrier system) is given by[5].

$$\sigma_{xx} = \sum_{i}^{N} \frac{n_i q_i \mu_i}{1+\mu_i^2 B^2} \quad (5)$$

$$\sigma_{xy} = \sum_{i}^{N} \frac{n_i q_i \mu_i^2 B}{1+\mu_i^2 B^2} \quad (6)$$

3. The fitting algroithm

If the number of carriers N is assumed to be known, experimental data for σ_{xx} and σ_{xy} can be fit to equations 5 and 6. This fit will be linear in the zero field conductivity ($n_i q_i \mu_i$) of each carrier and non-linear in the mobility (μ_i) of each carrier. The method used for the fit is as follows:

1. The carrier type (hole or electron) of each carrier is constrained to be fixed. This fixes the sign of the mobility of each carrier.

2. The zero field conductivity is constrained to be positive.

3. A sub-space of the N dimensional mobility space is selected for search. This sub-space is defined to be consistent with step 2. At each point in the sub-space a non-negative linear least squares fit is preformed and the chi-squared recorded.

4. The point in the sub-space with the minimum chi-squared is used as an initial guess for a non-linear least constrained least squares fit. The non-linear method with constraints on the mobility signs uses the damped Gauss-Newton method[2] and the Armijo-Goldstein step length principle.

4. Results

To provide a realistic test of the methods, Hall data on a sample was taken and an additional carrier was added to the data to test the algorithm's ability to find this low mobility, low conductivity carrier. Fits to the raw data for the sample showed two electron carriers with mobility of about 5000 and 2000. The conductivity of the added carrier was decreased, keeping the mobility fixed at 200, until the fits could not find the third carriers. In summary, the simplex fits with no constraints on the carrier signs could find the carriers to the 2% level, that is if the conductivity of the carrier was less than 2% of the conductivity of the other carriers the method did not find the carrier.

The method with mobility search was able to find a correct local minimum to the .5% level. However searches starting at this local minimum failed to improve the fit when the conductivity was less than 2%. The best method appears to be a search of mobility space with a non-negative least squares fit for the zero field conductivity at each point. The details of the test data are provided in table 1.

5. Conclusion

A method for find low mobility, low conductivity carriers in semi-conductors with parallel conduction, has been described. This method has been demonstrated to find carriers with conductivity of only .5% of the conductivity of the highest conductivity carrier in the material. The method requires the number of carriers and the carrier type (hole or electron) be known.

Table 1 Comparison of simplex fitting method and constrained sign method. Table entries are the mobility and zero field conductivity of third carrier added to measured hall data. Each row of the table is a decrease in the conductivity of the third carrier by a factor of four. The simplex method could not find a carrier of the correct sign for conductivity less than 2%, the constrained sign method found carriers to the .5% level.

	Simplex Method		Constrained signs		
Conductivity ratio	Mobility	Sheet Conductivity	Mobility	Sheet Conductivity	
	(cm^2/V-s)	(mho)	(cm^2/V-s)	(mho)	
1.000	195.50	0.3233	195.50	0.3233	
0.250	181.90	0.0820	181.90	0.0820	
0.063	125.50	0.0218	125.50	0.0218	
0.016			197.00	0.0049	Search only
0.004			197.00	0.0011	Search only

References

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